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On a class of T-η-hyperinvex function and generalized hyper differential dominated vector variational inequality problem.

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This paper is devoted to the study of a problem duality in a variational inequality to a complementarity problem. The concepts of $\widetilde{T}$-η-hyperinvex function and the generalized hyperdifferential dominated vector variational inequality problems are introduced. The author establishes an equivalent hyper problem theorem in ordered topological vector spaces modeled in a Hilbert space H. If $f \in L(X,Y)$ is a continuous map, and $\widehat{f} : X \to E$ is a lifting of f with respect to the covering map p ($p : E \to Y$ is a covering map such that $p^{-1}(Y_+)$ is nonempty in E, where $Y_+ = \{y : y \geq_Y 0\}$) and p is a complete hyper map on E, the equivalence between the expressions: 1° $\langle f, z\rangle \geq_Y 0$ for all $z \in X$ and, 2° $\langle \widehat{f}, z\rangle \geq_E 0$ for all $z \in X$ is proved. Main result: The equivalence between a generalized dominated differential variational inequality problem and a T-η-invex function in ordered topological vector spaces is studied. For a linear, monotonic and complete hyper map p on E (the ordered topological vector space), the proposed inequalities:

$$\begin{cases} \langle \nabla F(u), \eta(x,u)\rangle \geq_Y \langle T(u), \eta(x,u)\rangle & \text{for all } x, u \in X, \\ \langle \nabla \widetilde{F}(u), \eta(x,u)\rangle \geq_E \langle \widetilde{T}(u), \eta(x,u)\rangle & \text{for all } x, u \in X, \end{cases}$$

are equivalent. Furthermore, if X is an η-invex cone and $\widetilde{F}$ is $\widetilde{T}$-η-hyperinvex on X, then $p^{-1} \circ \widehat{p} \equiv 1_E$, the identity map on E, where $L(X,Y)$ is the set of continuous functionals from X to Y (or from X to E for $L(X,E)$) and $F : X \to Y$ is a continuous map, $\widetilde{F}$ is the lifting of F with respect to the covering map: $E \to Y$, where $p^{-1}(Y_+)$ is nonempty in E, $\eta : X \times X \to X$ is a vector valued map and $T : X \to L(X,Y)$ is any continuous map, $\widetilde{\nabla F} : X \to L(X,E) \equiv E$, $\widetilde{T} : X \to L(X,E) \equiv E$ are the lifting maps of $\nabla F : X \to L(X,Y)$ and $T : X \to L(X,Y)$ with respect to the same linear covering map $\widehat{p} : L(X,Y) \equiv E \to L(X,Y) \equiv Y$ respectively, and $\widehat{p}$ is the map defined by $\widetilde{p} \circ \langle \widetilde{T}(u), z\rangle = \langle \widetilde{p} \circ \widetilde{T}(u), z\rangle$. A proof of the existence theorem of $\widetilde{T}$-$\overline{\eta}$-hyperinvexivity of a shifting function in the presence of the generalized hyper differential dominated vector variational inequality problem by using a complete hyper map in the ordered topological vector space is proposed. *Jan Lovíšek (Bratislava)*

Keywords: reflexive Banach space; Hilbert space; variational inequality; generalization of convex function; vector valued map; T-η-invex function; $\widetilde{T}$-η-hyperinvex function; open hyper map; complete hyper map; η-invex cone; Hadamard derivative; Gâteaux derivative; Fréchet derivative