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**Complementary regions of knot and link diagrams.**

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The faces of a connected reduced link diagram divide the sphere into a collection of $n$-gons. This paper investigates which collections of $n$-gons can arise from link diagrams. In more detail, a strictly increasing sequence of integers $(a_1, a_2, a_3, \dots)$, with $a_1 \geq 2$, is said to be universal for knots and links if every knot and link has a reduced diagram in which each face is an $a_n$-gon for some $a_n$ in the sequence. (Note that not every $a_n$ in the sequence needs to be realized as an $a_n$-gon in the diagram.) It is shown that the following infinite sequences are universal for knots and links: $(3, 5, 7, 9, \dots)$, $(2, n, n+1, n+2, \dots)$ for each $n \geq 3$, and $(3, n, n+1, n+2, \dots)$ for each $n \geq 4$. In addition, it is shown that the finite sequences $(2, 4, 5)$, and $(3, 4, n)$ for $n \geq 5$ are universal for knots and links. Further results on the collection of $n$-gons that can arise from knot and link diagrams are given. For example, it is shown that every knot has a diagram with exactly two 3-gons with all other faces even; and it is shown that only sequences with a non-trivial common divisor $k$ arise from links with at least $k$ components. The paper concludes with a short list of questions.

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