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Enumerating $(2+2)$-free posets by the number of minimal elements and other statistics.

Discrete Appl. Math. 159, No. 17, 2098-2108 (2011).

Summary: An unlabeled poset is said to be $(2+2)$-free if it does not contain an induced subposet that is isomorphic to 2+2, the union of two disjoint 2-element chains. Let p_n denote the number of (2+2)-free posets of size n. In a recent paper, *M. Bousquet-Mélou* et al. [J. Comb. Theory, Ser. A 117, No. 7, 884–909 (2010; Zbl 1225.05026)] found, using the so called ascent sequences, the generating function for the number of (2+2)-free posets of size

$$n : P(t) = \sum_{n\geq 0} p_n t^n = \sum_{n\geq 0} \prod_{i=1}^{n} (1-(1-t)^i).$$

We extend this result in two ways. First, we find the generating function for (2+2-free posets when four statistics are taken into account, one of which is the number of minimal elements in a poset. Second, we show that if $p_{n,k}$ equals the number of (2+2)-free posets of size n with k minimal elements, then

$$P(t,z) = \sum_{n,k\geq 0} p_{n,k} t^n z^k = 1 + \sum_{n\geq 0} \frac{zt}{(1-zt)^{n+1}} \prod_{i=1}^{n} (1-(1-t)^i).$$

The second result cannot be derived from the first one by a substitution. Our enumeration results are extended to certain restricted permutations and to regular linearized chord diagrams through bijections in *M. Bousquet-Mélou* et al. [loc. cit.] and *A.Claesson*, *M. Dukes* and *S. Kitaev* [Australas. J. Comb. 49, 47–59 (2011; Zbl 1228.05232)]. Finally, we define a subset of ascent sequences counted by the Catalan numbers and we discuss its relations with (2+2)- and (3+1)-free posets.

Keywords: free posets; enumeration; generating functions; multiple-statistics; minimal elements; free posets
doi:10.1016/j.dam.2011.07.010