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Lattice based extended formulations for integer linear equality systems.

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Summary: We study different extended formulations for the set $X = \{\mathbf{x} \in \mathbb{Z}^n \mid \mathbf{A}\mathbf{x} = \mathbf{A}\mathbf{x}^0\}$ with $\mathbf{A} \in \mathbb{Z}^{m \times n}$ in order to tackle the feasibility problem for the set $X \cap \mathbb{Z}^n_+$. Pursuing the work of Aardal, Lenstra et al. using the reformulation $X = \{\mathbf{x} \in \mathbb{Z}^n \mid \mathbf{x} - \mathbf{x}^0 = \mathbf{Q}\lambda,\ \lambda \in \mathbb{Z}^{n-m}\}$, our aim is to derive reformulations of the form $\{\mathbf{x} \in \mathbb{Z}^n \mid \mathbf{P}(\mathbf{x} - \mathbf{x}^0) = \mathbf{T}\mu, \mu \in \mathbb{Z}^s\}$ with $0 \leq s \leq n - m$ where preferably all the coefficients of $\mathbf{P}$ are small compared to the coefficients of $\mathbf{A}$ and $\mathbf{T}$. In such cases the new variables μ appear to be good branching directions, and in certain circumstances permit one to deduce rapidly that the instance is infeasible. We give a polynomial time algorithm for identifying such $\mathbf{P}$, $\mathbf{T}$ if possible, and for the case that $\mathbf{A}$ has one row $\mathbf{a}$ we analyze the reformulation when $s = 1$, that is, one μ-variable is introduced. In particular, we determine the integer width of the extended formulations in the direction of the μ-variable, and derive a lower bound on the Frobenius number of $\mathbf{a}$. We conclude with some preliminary tests to see if the reformulations are effective when the number s of additional constraints and variables is limited.

Keywords: Lattice basis reduction; Integer width; Frobenius number
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