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Lipschitz functions have L_p-stable persistence.

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The results in this paper are motivated by the analysis of gene expression data during development. The biological data consists of a series of 13 measurements taken over the development stages of 14 cell types. For each cell type the 13 measurements can be viewed as a samples of a function $f : [0, 1] \to \mathbb{R}$. The underlying function is assumed to be Lipschitz. Each cross section represents a level surface. For the general results the authors consider a Lipschitz function $f : \mathbb{R} \to \mathbb{R}$. Let $\mathbb{X}_a = f^{-1}(-\infty, a]$ be the subset defined by a threshold a. Then as a increases the homology groups of $\mathbb{X}_a$ change. Thus one can observe homology classes appear and disappear. For a given homology class α one can define a point in $(d(\alpha), b(\alpha)) \in \mathbb{R}^2$, where $b(\alpha)$ is the birth level and $d(\alpha)$ is the death level. For a given ℓ the set $Dgm_\ell(f)$ of these points corresponding to ℓ-dimensional homology classes is called the ℓ-th persistence diagram of f. The Wasserstein distance between the diagrams for two functions f and g gives a measure of the (dis)-similarity between the two functions. The authors show a stability result for two Lipschitz functions on triangulable metric spaces that bounds the Wasserstein distance between them in terms of the L_∞ distance between them.

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