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Applications of combinatorial matrix theory to Laplacian matrices of graphs.
Discrete Mathematics and its Applications. Boca Raton, FL: CRC Press (ISBN 978-1-4398-6337-4/hbk). 405 p. £ 57.99 (2012).

The book under review is on the borderline between a graduate textbook and a research monograph on Laplacian matrices of graphs, focusing primarily on the algebraic connectivity and the Fiedler vector of graphs. The book owes its textbook appeal to detailed proofs, a large number of fully elaborated examples and observations, and a handful of exercises, making beginning graduate students as well as advanced undergraduates its primary audience. Still, it can serve as a useful reference book for experienced researchers as well. The book consists of a preface and eight chapters. It is not entirely self-contained, a prior knowledge of linear algebra at an undergraduate level is assumed. Chapter 1 introduces and discusses further concepts and results, mostly spectral ones: the spectral radius, the Perron-Frobenius theorem, the Geršgorin circles, doubly stochastic matrices and the Moore-Penrose and group inverses. Chapter 2 contains a short overview of the main graph theoretical concepts, followed by a detailed account of threshold graphs, important examples of graphs whose Laplacian spectra consist of integers only. Chapter 3 explains and motivates the study of Laplacian matrices by deriving them from continuous Laplacians, used in differential equations to study the energy flow through a region, and applying the discrete form of the Laplacian to minimize the energy of the representations of graph vertices in k-dimensional real spaces. It also contains a short study of Laplacian eigenvalues of random graphs, small world networks and scale-free networks. Chapter 4 is a survey of a number of results on the Laplacian spectrum: the effect of taking the unions, joins, products and complements of graphs on Laplacian spectrum, upper bounds on the spectral radius, the distribution of Laplacian eigenvalues less than, equal to and greater than one, as well as the recent proof of the Grone-Merris conjecture and the study of the Laplacian spectrum of threshold graphs. Chapter 5 is devoted to algebraic connectivity and its relation to the structure of a graph, and Chapters 6 and 7 are further devoted to the Fiedler vector of trees (Chapter 6) and of general graphs (Chapter 7). The main tool used in Chapters 6 and 7 are the bottleneck matrices, which are defined here as the inverse of the submatrix of L obtained by deleting the row and the column corresponding to u, for each vertex u, and whose spectral radius is closely related to algebraic connectivity. As a singular matrix, the Laplacian matrix has no usual inverse, and Chapter 8 is devoted to the group inverse of the Laplacian matrix, which relies heavily on the bottleneck matrices, and whose properties improve many of the earlier results in the book. *Dragan Stevanović (Niš)*

Keywords: Laplacian matrix; Laplacian matrices of graphs; algebraic connectivity; Fiedler vector; bottleneck matrices.