Smoothing SQP methods for solving degenerate nonsmooth constrained optimization problems with applications to bilevel programs.

Summary: We consider a degenerate nonsmooth and nonconvex optimization problem for which the standard constraint qualification such as the generalized Mangasarian-Fromovitz constraint qualification (GMFCQ) may not hold. We use smoothing functions with the gradient consistency property to approximate the nonsmooth functions and introduce a smoothing sequential quadratic programming (SQP) algorithm under the $l_\infty$ penalty framework. We show that any accumulation point of a selected subsequence of the iteration sequence generated by the smoothing SQP algorithm is a Clarke stationary point, provided that the sequence of multipliers and the sequence of penalty parameters are bounded. Furthermore, we propose a new condition called the weakly generalized Mangasarian-Fromovitz constraint qualification (WGMFCQ) that is weaker than the GMFCQ. We show that the extended version of the WGMFCQ guarantees the boundedness of the sequence of multipliers and the sequence of penalty parameters and thus guarantees the global convergence of the smoothing SQP algorithm. We demonstrate that the WGMFCQ can be satisfied by bilevel programs for which the GMFCQ never holds. Preliminary numerical experiments show that the algorithm is efficient for solving degenerate nonsmooth optimization problems such as the simple bilevel program.

Keywords: nonsmooth optimization; constrained optimization; smoothing function; sequential quadratic programming algorithm; bilevel program; constraint qualification

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