
io-port 05815439**Cohen, Albert; DeVore, Ronald; Schwab, Christoph****Convergence rates of best N -term Galerkin approximations for a class of elliptic SPDEs.**

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Summary: Deterministic Galerkin approximations of a class of second order elliptic PDEs with random coefficients on a bounded domain $D \subset \mathbb{R}^d$ are introduced and their convergence rates are estimated. The approximations are based on expansions of the random diffusion coefficients in $L^2(D)$ -orthogonal bases, and on viewing the coefficients of these expansions as random parameters $y = y(\omega) = (y_i(\omega))$. This yields an equivalent parametric deterministic PDE whose solution $u(x, y)$ is a function of both the space variable $x \in D$ and the in general countably many parameters y . We establish new regularity theorems describing the smoothness properties of the solution u as a map from $y \in U = (-1, 1)^\infty$ to $V = H_0^1(D)$. These results lead to analytic estimates on the V norms of the coefficients (which are functions of x) in a so-called “generalized polynomial chaos” (gpc) expansion of u . Convergence estimates of approximations of u by best N -term truncated V valued polynomials in the variable $y \in U$ are established. These estimates are of the form N^{-r} , where the rate of convergence r depends only on the decay of the random input expansion. It is shown that r exceeds the benchmark rate $1/2$ afforded by Monte Carlo simulations with N “samples” (i.e., deterministic solves) under mild smoothness conditions on the random diffusion coefficients. A class of fully discrete approximations is obtained by Galerkin approximation from a hierarchic family $\{V_l\}_{l=0}^\infty \subset V$ of finite element spaces in D of the coefficients in the N -term truncated gpc expansions of $u(x, y)$. In contrast to previous works, the level l of spatial resolution is adapted to the gpc coefficient. New regularity theorems describing the smoothness properties of the solution u as a map from $y \in U = (-1, 1)^\infty$ to a smoothness space $W \subset V$ are established leading to analytic estimates on the W norms of the gpc coefficients and on their space discretization error. The space W coincides with $H^2(D) \cap H_0^1(D)$ in the case where D is a smooth or convex domain. Our analysis shows that in realistic settings a convergence rate N_{dof}^{-s} in terms of the total number of degrees of freedom N_{dof} can be obtained. Here, the rate s is determined by both the best N -term approximation rate r and the approximation order of the space discretization in D .

Keywords: stochastic and parametric elliptic equations; Wiener polynomial chaos; approximation rates; nonlinear approximation; sparsity
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