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Compact CCC spaces of prescribed density via hypergraphs.

Miklós, D. (ed.) et al., Combinatorics, Paul Erdős is eighty. Vol. 1. Budapest: János Bolyai Mathematical Society. Bolyai Society Mathematical Studies. 239-252 (1993).

The negative partition relation $\kappa \nrightarrow [\kappa]^{<\omega}_{\kappa}$ means that there is a coloring $c : [\kappa]^{<\omega} \to \kappa$ such that for every $X \in [\kappa]^{\kappa}$ and $\xi < \kappa$ there is $F \in [X]^{<\kappa}$ such that $c(F) = \xi$. Now (reviewer's notation) let $\mathcal{C}$ be the class of compact Hausdorff spaces with the countable chain condition. The authors prove this result in ZFC: If κ is regular and satisfies $\kappa \nrightarrow [\kappa]^{<\omega}_{\kappa}$, then some $X \in \mathcal{C}$ satisfies $d(X) = \kappa$. When (regular) κ is not the successor of a singular cardinal, the space $X \in \mathcal{C}$ with $d(X) = \kappa$ may be chosen even to have property (K) – i.e., of any $\aleph_1$-many open sets in X, some $\aleph_1$-many have nonempty pairwise intersections. It follows in particular that for $\kappa \in \{\omega_1, \omega_2, \mathfrak{c}\}$ there is $X_\kappa \in \mathcal{C}$ with $d(X_\kappa) = \kappa$. It is well known, further, that for $\kappa \geq \omega$ the space $K_\kappa = \{0,1\}^\kappa$ satisfies $K_\kappa \in \mathcal{C}$, $d(K_\kappa) = \log \kappa$. According to the authors it remains unknown, however, whether every $\kappa \geq \omega$ has the form $\kappa = d(X)$ for some $X \in \mathcal{C}$. *W.W.Comfort (Singapur)*

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