
io-port 05917130**Galeana-Sánchez, H.; Silva, Raúl González****Grundy functions in the Cartesian product.**

AKCE Int. J. Graphs Comb. 7, No. 1, 11-27 (2010).

Summary: Let D be a digraph, $\alpha = (\alpha_v)_{v \in V(D)}$ a family where the α_v are mutually disjoint digraphs. The Cartesian product of α over D , denoted by $\sigma(D, \alpha)$ is defined as follows: (i) $V(\sigma(D, \alpha)) = \bigcup_{v \in V(D)} V(\alpha_v)$, (ii) $A(\sigma(D, \alpha)) = \bigcup_{v \in V(D)} A(\alpha_v) \cup \{(x, y) \mid x \in \alpha_u, y \in \alpha_v \text{ and } (u, v) \in A(D)\}$. A nonnegative integer function $g(x)$ is called a Grundy function on G if for every vertex x , $g(x)$ is the smallest nonnegative integer which does not belong to the set $\{g(y) \mid y \in \Gamma^+(x)\}$. This concept was originated by *P. M. Grundy* in [Mathematics and games, Eureka 2, 6–8 (1939)], for digraphs without directed cycles. It was extended by *C. Berge* and *M. P. Schützenberger* in [C. R. Acad. Sci., Paris 242, 1672–1674 (1956; Zbl 0075.29601)]. Also it was diffused by *C. Berge* in [C. R. Acad. Sci., Paris 242, 1404–1407 (1956; Zbl 0075.29602)]. A set $I \subseteq V(D)$ is independent if $A(D[I]) = \emptyset$. A kernel N of D is an independent set of vertices such that for each $z \in V(D) - N$ there exists a zN -arc. A digraph D is a kernel-perfect digraph whenever each one of its induced subdigraphs has a kernel. The concepts of kernel of a digraph and Grundy function of a digraph are nearly related. In this paper we prove sufficient and necessary conditions for the Cartesian product $\sigma(D, \alpha)$ of a family of digraphs $\alpha = (\alpha_v)_{v \in V(D)}$ over a digraph D to have a Grundy function in terms of the existence of Grundy function or kernel in D and in each α_v . Also it is shown a relationship between the size of the Grundy function obtained for $\sigma(D, \alpha)$ and the size of the Grundy functions of the factors α_v .

Keywords: Grundy function; Cartesian product; kernel; kernel-perfect digraph