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io-port 06036385

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Minimal enclosing discs, circumcircles, and circumcenters in normed planes. I.

Comput. Geom. 45, No. 5-6, 258-274 (2012).

Let $\|\cdot\|$ be a norm for the vector space $\mathbb{R}^2$, and suppose we are given three non–colinear points t_1, t_2, t_3 in the plane. In contrast to the Euclidean case, the intersection of two circles may contain line segments, and there may be multiple circumcircles that contain the points t_1, t_2, t_3. In this article, the authors classify all possible intersections of two circles in the plane. This classification is then used to explicitly determine the regions in the plane that consist of points x for which there exists a norm such that x is the center of a circumcircle of t_1, t_2, t_3. Moreover for the given norm $\|\cdot\|$, in the case where the line segment from t_1 to t_2 lies in the intersection of two distinct circumcircles $\mathcal{C}, \mathcal{C}'$ with centers c, c', the authors determine further restrictions on the regions where the centers can be located; in particular, it is shown that $\mathcal{C}, \mathcal{C}'$ have the same radius if and only if c, c' lie on the line passing through the midpoint of t_1 and t_3 and the midpoint of t_2 and t_3. The article is largely self-contained and intended for a general mathematical audience.

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Keywords: circumcenters; intersection of norm circles; minimal enclosing balls; Minkowski geometry; normed plane

doi:10.1016/j.comgeo.2012.01.007