The number of rational points of a family of hypersurfaces over finite fields.


Summary: Let $\mathbb{F}_q$ denote the finite field of odd characteristic $p$ with $q$ elements ($q = p^e, e \in \mathbb{N}$) and $\mathbb{F}_q^*$ represent the nonzero elements of $\mathbb{F}_q$. In this paper, by using the Smith normal form of the index matrix, we give a formula for the number of rational points of the following family of hypersurface over $\mathbb{F}_q$:

$$
\sum_{j=0}^{t-1} \sum_{i=1}^{r_j} a_{r_j+i} x_1^{e_j+1} \cdots x_{n_j+1}^{e_j+1,n_j+1} - b = 0,
$$

where the integers $t > 0$, $r_0 = 0 < r_1 < r_2 < \ldots < r_t$, $n_1 < n_2 < \ldots < n_t$, $b \in \mathbb{F}_q$, $a_i \in \mathbb{F}_q^*$ ($i = 1, \ldots, r_t$), and the index of each variable is a positive integer. Especially under some certain conditions, we get an explicit formula of the number of rational points of the above hypersurface. This generalizes greatly the results obtained by Wolfmann in 1994, Sun in 1997 and Wang and Sun in 2005, respectively.

Keywords: hypersurface; rational point; finite field; Smith normal form; system of linear congruences

doi:10.1016/j.jnt.2015.04.006