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Shiromoto, Keisuke
Singleton bounds for codes over finite rings.
J. Algebr. Comb. 12, No.1, 95-99 (2000).

Let R be a finite commutative quasi-Froebenius (QF) ring and let $V = R^n$ be the free module of rank n consisting of all n-tuples of elements of R. A code C of length n over R is an R-submodule of V. For every $x = (x_1, x_2, \ldots, x_n) \in V$ and $r \in R$, the complete weight of x is defined by $n_r(x) = |\{i \mid x_i = r\}|$. Let $a_0 = 0$ and a_r be a positive real number for any nonzero $r \in R$. A general weight function $w(x)$ is defined by

$$w(x) = \sum_{r \in R} a_r n_r(x).$$

For any linear $[n, k, d]$ code over a finite field, the Singleton bound says that $d \leq n - k + 1$. The main result here is the following generalization of the Singleton bound for a general weight function over R. Theorem. Let C be a code with minimum distance d over a finite commutative QF ring R. Let $w(x)$ be a general weight function on C. Then

$$\big[(d-1)/A\big] \leq n - \log_{|R|} |C|,$$

where $A = \max\{a_r \mid r \in R\}$ and $[b]$ is the integer part of b. Finally, the author presents some applications of his result to codes over $\mathbb{Z}_l$ (and in particular $\mathbb{Z}_4$) for the special weight functions corresponding to the Hamming, Lee and Euclidean weights. *T.Helleseth (Bergen)*

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