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Finite locally-quasiprimitive graphs.

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A quasiprimitive permutation group contains no non-trivial non-transitive normal subgroups. A finite graph is said to be locally quasiprimitive with respect to a subgroup of automorphisms G if each vertex-stabilizer G_α acts quasiprimitively on the neighbbors of α. An infinite family of locally-quasiprimitive but not locally-primitive graph (with respect to the full automorphism group) based on $\mathrm{PSL}(2,p)$s is constructed in the paper; hence, the concept of local quasiprimitivity is shown to be weaker than the concept of local primitivity. The family of locally-quasiprimitive graphs is shown to be closed under the formation of the so-called normal quotients, induced by a normal subgroup, and each locally-quasiprimitive graph is shown to be a multicover of each of its normal quotients. This allows for the concept of minimal locally-quasiprimitive graphs – the "building stones" of all graphs in this class – which gives hope for possible classification of all locally-quasiprimitive graphs. Both the structure of these minimal graphs as well as the process of obtaining all locally-quasiprimitive graphs whose normal quotient is a given locally-quasiprimitive graph are considered. To facilitate the second task, the concept of an extender of a locally-quasiprimitive graph is introduced, and the simple G-admissible covers of such an extender are shown to correspond to locally-quasiprimitive normal multicovers of the original graph. The paper closes with some partial results on the problem of reconstructing a locally-quasiprimitive graph from its normal quotients, and with an extensive list of open problems. *Robert Jajcay (Terre Haute)*

Keywords: arc-transitive graph; quasiprimitive action; automorphism group; stabilizer; normal quotient
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