
io-port 01945006**Green, R.M.****On rank functions for heaps.**

J. Comb. Theory, Ser. A 102, No.2, 411-424 (2003).

Summary: In this paper there are three “players”; (ranked) (labeled) posets, properties in this review taken to be known; Coxeter groups, properties taken to be known also; and (labeled) heaps, derived from labeled posets (E, \leq) via maps $\varepsilon : E \rightarrow P$, where P is a set equipped with C , a symmetric binary (concurrency) relation which yields a group Γ with edges xy iff $x \neq y$ and xCy such that the two axioms (1) $\varepsilon(\alpha)C\varepsilon(\beta)$ implies α and β are related in (E, \leq) and (2) \leq is the transitive closure of \leq_c where $\alpha \leq_c \beta$ iff $\alpha \leq \beta$ and $\varepsilon(\alpha)C\varepsilon(\beta)$. Since labeled posets are trivially labelled heaps if $\varepsilon(\alpha) = \alpha$ and $\alpha C \beta$ precisely when $\alpha \leq \beta$ or $\beta \leq \alpha$, one may consider “heap theory” as an extension of “labeled poset theory”. Gathering labeled heaps into isomorphism classes, $(E, \leq, \varepsilon) \cong (E', \leq', \varepsilon')$ provided $\varphi : E \rightarrow E'$ is an isomorphism of posets and $\varepsilon = \varepsilon' \circ \varphi$, the embedding $\varepsilon : E \rightarrow P$ generates a subgraph of Γ , i.e. the full subgraph on $\varepsilon(E)$, the concurrency subgraph. If the latter is acyclic then it is shown that (Theorem 2.1.1) the conditions: (1) E is ranked; (2) every subinterval of E is ranked; (3) every minimal balanced subinterval of E is ranked; are indeed equivalent, providing a generalization of the standard theorem which is included via the identity embedding on E . The introduction of the second player and its known connections with the first player suggests that it may be profitable to take a “heap” viewpoint, which proves to be the case. The resulting Theorem (3.2.3) essentially states (and proves) a necessary and sufficient condition for the heap of a fixed fully commutative element of an FC-finite Coxeter group to be ranked, in terms of Theorem 2.1.1 and a particular labeling associated with the group. *Joseph Neggers (Tuscaloosa)*

Keywords: heaps of pieces; ranked posets; Coxeter groups; labeled posets; labeled heaps; concurrency
doi:10.1016/S0097-3165(03)00055-4