io-port 02147023

Volkov, M.V.

Reflexive relations, extensive transformations and piecewise testable languages of a given height.

Int. J. Algebra Comput. 14, No. 5-6, 817-827 (2004).

Let Σ be a finite alphabet and Σ^* be the free monoid generated by Σ. A language over Σ is 'piecewise testable of height k' if it belongs to the Boolean algebra generated by languages of the form $\Sigma^* x_1 \Sigma^* \cdots x_m \Sigma^*$ where $0 \le m \le k$ and $x_1, \ldots, x_m \in \Sigma$. Simon gave an elegant algebraic characterization of the piecewise testable languages. Indeed, a language is piecewise testable if and only if it can be recognized by a finite $\mathcal{J}$-trivial monoid (a monoid M is $\mathcal{J}$-trivial if $MaM = MbM$ implies $a = b$ for all $a, b \in M$). By $\mathcal{R}_n$ is denoted the monoid of all reflexive binary relations on a set with n elements. It can be thought of as a submonoid of the monoid of all $n \times n$ matrices over the Boolean semiring $\mathcal{B} = \langle \{0,1\}; +, \cdot \rangle$, that is, the submonoid consisting of matrices in which all diagonal entries are 1. By $\mathcal{U}_n$ is denoted the submonoid of $\mathcal{R}_n$ consisting of upper triangular matrices, and by $\mathcal{C}_n$ the monoid of all order preserving and extensive transformations of a chain with n elements. The class $\mathbf{J}$ of all finite $\mathcal{J}$-trivial monoids forms a pseudovariety generated by each of the three sequences $\{\mathcal{U}_n\}$, $\{\mathcal{R}_n\}$ and $\{\mathcal{C}_n\}$ $(n = 1, 2, \ldots)$. For every $k = 1, 2, \ldots$, the class of all piecewise testable languages of height k corresponds to some pseudovariety of finite monoids denoted by $\mathbf{J}_k$. The union of the increasing sequence $\mathbf{J}_1 \subset \mathbf{J}_2 \subset \cdots \subset \mathbf{J}_k \subset \cdots$ is $\mathbf{J}$. In this paper, the author shows that for every k, each of $\mathcal{R}_{k+1}$, $\mathcal{U}_{k+1}$, $\mathcal{C}_{k+1}$ generates $\mathbf{J}_k$. Using this and some results of Blanchet-Sadri, the author gives a complete solution of the finite basis problem for the monoids $\mathcal{U}_n$, $\mathcal{R}_n$ and $\mathcal{C}_n$ for each n (also called Straubing's monoids). *Francine Blanchet-Sadri (Greensboro)*

Keywords: piecewise testable languages; $\mathcal{J}$-trivial monoids; pseudovarieties of finite monoids; reflexive relations; order preserving transformations; extensive transformations; finite basis problem
doi:10.1142/S0218196704002018