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Improved spectral sparsification and numerical algorithms for SDD matrices.

Dürr, Christoph (ed.) et al., STACS 2012. 29th international symposium on theoretical aspects of computer science, Paris, France, February 29th – March 3rd, 2012. Wadern: Schloss Dagstuhl – Leibniz Zentrum für Informatik (ISBN 978-3-939897-35-4). LIPICS – Leibniz International Proceedings in Informatics 14, 266-277, electronic only (2012).

Summary: We present three spectral sparsification algorithms that, on input a graph G with n vertices and m edges, return a graph H with n vertices and $O(n \log n/\varepsilon^2)$ edges that provides a strong approximation of G. Namely, for all vectors x and any $\varepsilon > 0$, we have

$$(1-\varepsilon)x^T L_G x \leq x^T L_H x \leq (1+\varepsilon)x^T L_G x,$$

where L_G and L_H are the Laplacians of the two graphs. The first algorithm is a simple modification of the fastest known algorithm and runs in $\tilde{O}(m \log^2 n)$ time, an $O(\log n)$ factor faster than before. The second algorithm runs in $\tilde{O}(m \log n)$ time and generates a sparsifier with $\tilde{O}(n \log^3 n)$ edges. The third algorithm applies to graphs where $m > n \log^5 n$ and runs in $\tilde{O}(m \log_{m/n \log^5 n} n)$ time. In the range where $m > n^{1+r}$ for some constant r this becomes $\tilde{O}(m)$. The improved sparsification algorithms are employed to accelerate linear system solvers and algorithms for computing fundamental eigenvectors of dense SDD matrices.

Keywords: spectral sparsification; linear system solving
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