Optimal point movement for covering circular regions.

Summary: Given $n$ points in a circular region $C$ in the plane, we study the problems of moving the $n$ points to the boundary of $G$ to form a regular $n$-gon such that the maximum (min-max) or the sum (min-sum) of the Euclidean distances traveled by the points is minimized. These problems have applications, e.g., in mobile sensor barrier coverage of wireless sensor networks. The min-max problem further has two versions: the decision version and the optimization version. For the min-max problem, we present an $O(n \log^2 n)$ time algorithm for the decision version and an $O(n \log^3 n)$ time algorithm for the optimization version. The previously best algorithms for the two problem versions take $O(n^{3.5})$ time and $O(n^{3.5} \log n)$ time, respectively. For the min-sum problem we show that a special case with all points initially lying on the boundary of the circular region can be solved in $O(n^2)$ time, improving a previous $O(n^4)$ time solution. For the general min-sum problem, we present a 3-approximation $O(n^2)$ time algorithm. In addition, a by-product of our techniques is an algorithm for dynamically maintaining the maximum matching of a circular convex bipartite graph; our algorithm can handle each vertex insertion or deletion on the graph in $O(\log^2 n)$ time. This result may be interesting in its own right.

Keywords: computational geometry; algorithms and data structures; circular region coverage; barrier coverage; mobile sensors; dynamic maximum matching; circular convex bipartite graph
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