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Multivariate discrete least-squares approximations with a new type of collocation grid.

Summary: In this work, we discuss the problem of approximating a multivariate function by discrete least-squares projection onto a polynomial space using a specially designed deterministic point set. The independent variables of the function are assumed to be random variables, stemming from the motivating application of uncertainty quantification. Our deterministic points are inspired by a theorem due to André Weil. We first work with the Chebyshev measure and consider the approximation in Chebyshev polynomial spaces. We prove the stability and an optimal convergence estimate, provided the number of points scales quadratically with the dimension of the polynomial space. A possible application for quantifying epistemic uncertainties is then discussed. We show that the point set asymptotically equidistributes to the product-Chebyshev measure, allowing us to propose a weighted least-squares framework and extending our method to more general polynomial approximations. Numerical examples are given to confirm the theoretical results. It is shown that the performance of our deterministic points is similar to that of randomly generated points. However, our construction, being deterministic, does not suffer from probabilistic qualifiers on convergence results (e.g., convergence “with high probability”).

Keywords: uncertainty quantification; discrete least-squares approximations; Chebyshev polynomials; stability
doi:10.1137/130950434