Butterfly factorization.

Summary: The paper introduces the butterfly factorization as a data-sparse approximation for the matrices that satisfy a complementary low-rank property. The factorization can be constructed efficiently if either fast algorithms for applying the matrix and its adjoint are available or the entries of the matrix can be sampled individually. For an $N \times N$ matrix, the resulting factorization is a product of $O(\log N)$ sparse matrices, each with $O(N)$ nonzero entries. Hence, it can be applied rapidly in $O(N \log N)$ operations. Numerical results are provided to demonstrate the effectiveness of the butterfly factorization and its construction algorithms.

Keywords: data-sparse matrix; butterfly algorithm; randomized algorithm; matrix factorization; operator compression; Fourier integral operators; special functions

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