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Pride, Stephen J.

Homological finiteness conditions for groups, monoids, and algebras.

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Let KB be a monoid ring over a commutative ring K and a monoid B. Consider K as a left KB-module with the KB-action $k_1b \cdot k_2 = k_1k_2$. Then B is said to be of type 'left-FP_n' (over K) if there is a partial free resolution $0 \leftarrow {}_BK \leftarrow P_0 \leftarrow P_1 \leftarrow \cdots \leftarrow P_n$, where P_i are finitely generated free left KB-modules. The author calls a semigroup to be of type 'weak bi-FB_n' if it has this property with respect of partial free resolutions of (KB, KB)-bimodules (K is considered then as the (KB, KB)-bimodule ${}_BK_B$). A semigroup B is called of type 'bi-FP_n' if there exists a partial resolution $0 \leftarrow KB \leftarrow F_0 \leftarrow F_1 \leftarrow \cdots \leftarrow F_n$ of (KB, KB)-bimodules where F_i are finitely generated free. It is proved that 1) if a monoid is weak bi-FB_n, then it is both left- and right-FB_n; 2) If a group or a connected graded algebra is left-FP_n (or, equivalently, right-FP_n), then it is bi-FP_n (a graded algebra $A = \bigoplus_{i\geq 0} A_i$ is called 'connected' if A_0 has basis the identity 1_A); 3) The properties left-FP_n, right-FP_n, bi-FP_n and weak bi-FP_n are closed under retractions.

Peeter Normak (Tallinn)

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