Vaught’s theorem on axiomatizability by a scheme.

Vaught’s set theory $\mathbf{VS}$ is the theory (in the language with $=$ and $\in$) with the axioms

$$\forall x_0, \ldots, x_{n-1} \exists y \forall u (u \in y \iff \bigvee_{i<n} u = x_i)$$

for $n = 0, 1, 2, \ldots$.

A Vaught theory is a theory that directly interprets (that is, the interpretation is not relativized and translates identity to identity) $\mathbf{VS}$. Vaught’s theorem says: all recursively enumerable Vaught theories are axiomatizable by a scheme. The theory of unordered pairing $\mathbf{VS}_2$ is the theory (in the language with $=$ and $\in$) with (essentially) the axiom $\forall x_0, x_1 \exists y \forall u (u \in y \iff u = x_0 \lor u = x_1)$. A pair theory is a theory that directly interprets $\mathbf{VS}_2$. The main theorem of the paper under review says: all recursively enumerable pair theories are axiomatizable by a scheme. Visser’s theorem strictly improves Vaught’s theorem (because there are consistent decidable pair theories but all consistent Vaught theories are essentially undecidable).

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